LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2014**

ST 1820 - ADVANCED DISTRIBUTION THEORY

Date : 31/10/2014 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

SECTION - A

Answer ALL questions. Each carries TWO marks: (10 x 2 = 20 marks)

1. Give an example for a random variable and show that it satisfies the definition of a random variable.
2. Define distribution function of a random variable and write its properties.
3. Show that two random variables X and Y which are not identical can have identical distributions.
4. State the pdf of truncated binomial left truncated at '0' and obtain its mgf.
5. Find the pdf of (i) truncated exponential distribution, left truncated at '2', (ii) truncated exponential distribution right truncated at '10'.
6. Consider iid non-negative and integer valued random variables $X_1, X_2, \ldots, X_n$. Show that X_1 is geometric, when $X_{(1)} = \text{Min}\{X_1, X_2, \ldots, X_n\}$ is a geometric random variable.
7. Obtain the pgf of a log-series distribution and hence find its mgf.
8. Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Obtain the two marginal distributions.
9. Let $(X_1, X_2) \sim BVP(\lambda_1, \lambda_2, \lambda_{12})$. Prove that X_1 is independent of X_2 if and only if $\lambda_{12} = 0$.
10. Define compound distribution of X, when (i) θ is discrete, (ii) θ is continuous.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks: (5 x 8 = 40 marks)

11. Consider the distribution function of a random variable X given by

$$\mathbf{F(x)} = \begin{cases} 0, & x < -1 \\ \frac{(x+2)}{4}, & -1 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

Find (i) the decomposition of F, (ii) mgf of F.

12. Derive the mean, variance and mgf of truncated Poisson distribution, left truncated at '0'.
13. State and prove a characterization of Bernoulli distribution through moments.
14. Consider iid Poisson random variables X_1 and X_2 with parameter λ. Obtain the conditional distribution of X_1 given $X_1 + X_2 = n$.
15. State and prove the recurrence relation for the raw moments of a log-series distribution.
16. State and establish Skitovitch theorem regarding normal distributions.
17. Consider independent normal variables X_1, X_2, and X_3 with $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$ and $V(X_3) = 3$. Check whether or not the following pairs are independent:
    (i) $X_1 + X_2$ and $X_1 – X_2$
    (ii) $X_1 + X_2 - 2X_3$ and $X_1 – X_2$
    (iii) $2X_1 + X_3$ and $X_2 – X_3$.
18. Derive the mgf of inverse Gaussian distribution.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks: (2 x 20 = 40 marks)

19(a) Consider a non-negative integer valued random variable X which satisfies lack of memory property. Find the distribution of X. (10)

(b) Obtain the pgf, mgf and mean of power-series distribution. Hence find the pgf, mgf and mean of binomial distribution. (10)

20(a) Derive the relationship among mean, median and mode of log-normal distribution. (10)

(b) Let $X_1 \sim G(\alpha, p_1)$, $X_2 \sim G(\alpha, p_2)$ and $X_1 \perp\!\!\!\perp X_2$. Find the distribution of (i) $X_1 + X_2$, (ii) $X_1 / (X_1 + X_2)$. Hence verify the independence of $X_1 + X_2$ and $(X_1 / (X_1 + X_2))$. (10)

21(a) Let $X_1, X_2, \ldots , X_n$ be independent random variables such that $X_r \sim IG(\mu_r, \lambda_r)$, $r = 1, 2, \ldots , n$. Derive the distribution of $\sum_{r=1}^{n}((\lambda_r \quad \mu_r)/\mu_r^2)$. (8)

(b) Let $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2 , \sigma_2^2 , \rho)$. Find the two marginal distributions. (12)

22(a) Obtain the mgf $M(t_1, t_2)$ of $BVN(\mu_1, \mu_2, \sigma_1^2 , \sigma_2^2 , \rho)$. (10)

(b) Define non-central t – distribution and find its pdf. (10)

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